POLARIZATION EXTENSIONS OF THE MONOSTATIC-BISTATIC EQUIVALENCE THEOREM

JANUARY 1966

S. H. Bickel

Prepared for

DIRECTORATE OF RADAR AND OPTICS

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
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L. G. Hanscom Field, Bedford, Massachusetts

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ABSTRACT

For convex non-depolarizing bodies, the monostatic-bistatic approximation is exact to first order terms in the bistatic angle. The second order error effect causes widening of the lobe structure with increasing bistatic angle. For depolarizing bodies, the error is again second order in the bistatic angle provided that the depolarizing edges have parallel orientation. Consequently, the theorem can be extended to convex bodies of revolution.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

HARRY M. BYRAM
ESD Project Officer
Project 750

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SECTION I

MONOSTATIC-BISTATIC EQUIVALENCE FOR POINT SCATTERERS

The monostatic-bistatic theorem states that the voltage which is developed at the receiver terminals of a bistatic radar is the same as that which would be developed at the receiver if both the transmitting and receiving antenna are located on the bisector of the bistatic angle ξ.. See Figure 1.
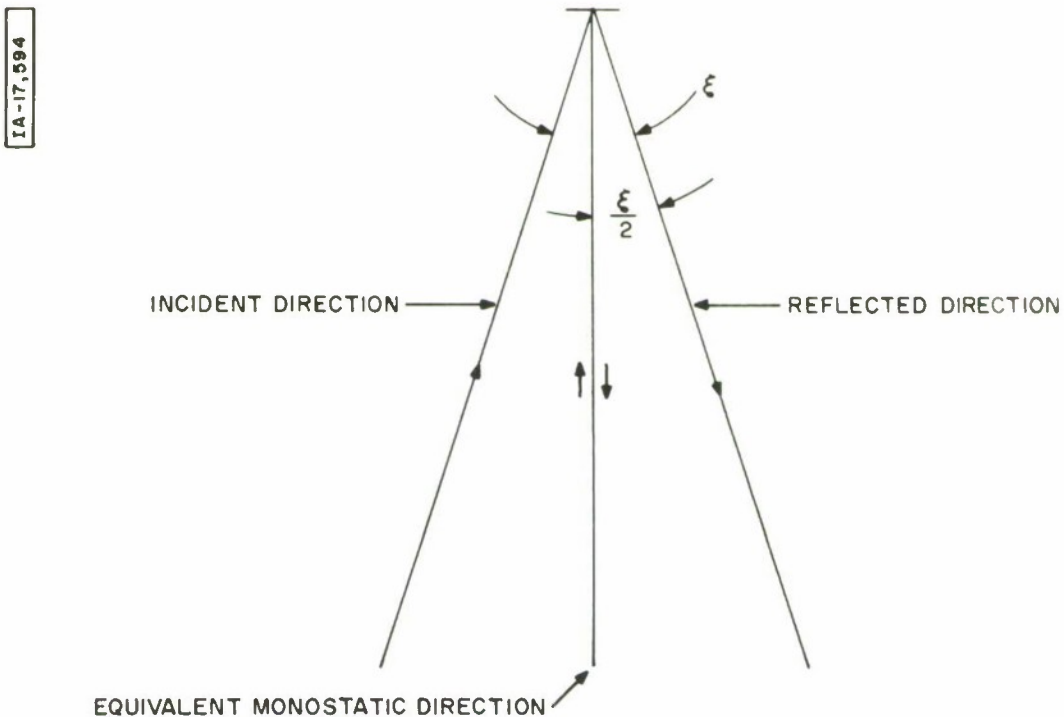
Figure 1.

The theorem is most simply demonstrated from Snell's law for speculars. Here the angle of incidence is equal to the angle of specular reflection so that the monostatic equivalent radar sees a specular whenever the bistatic radar sees one.

In order to extend the monostatic-bistatic theorem beyond specular returns, consider a set of n point scatterers and let the vector $\bar{\rho}_i$ designate the location of the i'th scatterer from the point 0 and let $\hat{r}_1$ and $\hat{r}_2$ be unit vectors in the direction of incidence and observation. See Figure 2. Neglecting interaction

Figure 2.

between the scatterers, then at large distances the field scattered in the direction of observation is proportional to

$$E_B = \sum_{i=1}^{n} e^{jk\bar{\rho}_i \cdot \left(\hat{r}_1 + \hat{r}_2 \right)} \, ,$$

(1)

where:

$$k = \frac{\omega}{c} \, .$$

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For the monostatic return, the direction of incidence and observation coincide. Setting $\hat{r}_1 = \hat{r}_2 = \hat{r}$, Equation (1) for the monostatic return becomes

$$E_M = \sum_{i=1}^{n} e^{j2k\bar{\rho}_i \cdot \hat{r}} \, .$$

(2)

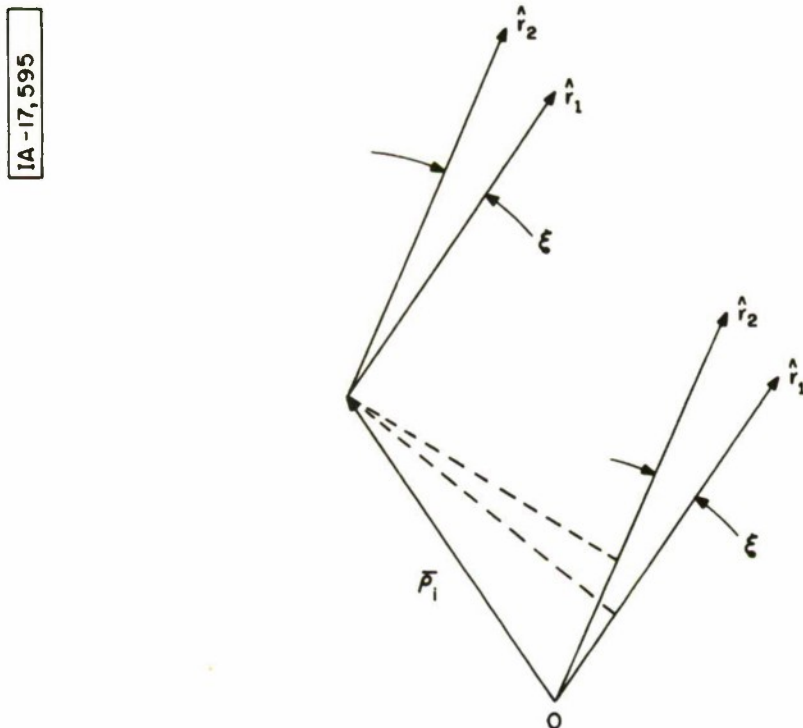Now take the vector $\hat{r}$ to be in the direction of the bisector of the bistatic angle ξ defined by $\hat{r}_1$ and $\hat{r}_2$ in Figure 2. Hence,

$$\hat{r} = \frac{\hat{r}_1 + \hat{r}_2}{\left| \hat{r}_1 + \hat{r}_2 \right|} = \frac{\hat{r}_1 + \hat{r}_2}{2 \cos \frac{\xi}{2}} \, .$$

(3)

Substitution of Equation (3) into (1) yields

$$E_B = \sum_{i=1}^{n} e^{j2\left(k \cos \frac{\xi}{2}\right) \bar{\rho}_i \cdot \hat{r}} \, .$$

(4)

Comparing Equation (2) with (4), one finds that the bistatic return is identical to the monostatic return on the bisector of the bistatic angle if the monostatic frequency is given by

$$\omega_M = \omega \cos \frac{\xi}{2} \, .$$

(5)

Although this frequency shift is second order in the bistatic angle, it may or may not be important depending upon the size of the body in wavelengths and the region where the monostatic-bistatic approximation is being used. The effect will tend to be small near speculars and to increase in the regions of the sidelobes.

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This can be demonstrated by considering monostatic scattering from a line scatterer of length L. The location of the n'th null in the scattering pattern is given approximately by

$$\theta_n \approx \frac{n\lambda}{2L} \approx n\theta_o , \tag{6}$$

where θ_o is the 3-db beamwidth and is given approximately by $\frac{\lambda}{2L}$. From Equation (5) frequency is shifted for the bistatic return so that the error in the location of the nulls in beamwidths is given by

$$\delta = \frac{\theta_{n'} - \theta_n}{\theta_o} = n \frac{\lambda' - \lambda}{\lambda} \approx \frac{n\xi^2}{8} . \tag{7}$$

Since the error is second order in ξ, then for small ξ the shifting of the lobe structure is negligible for the sidelobes near the specular. Figure 3 illustrates the broadening of the lobe pattern and the resulting shifts in the null locations for a fixed bistatic angle.

Equation (7) can also be used to establish the maximum permissible bistatic angle for a given error in the location of the n'th null in the scattering pattern. Suppose that the maximum tolerable error is 1/16 of a beamwidth. In this case, restriction on the bistatic angle is

$$\xi \leq \frac{1}{\sqrt{2n}} . \tag{8}$$

For example, a pattern with 20 lobes restricts the bistatic angle to 10 degrees.

The results of analysis of the far field scattering in situations where physical optics applies (i.e., the total magnetic field is approximated by twice

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BISTATIC PATTERN ———————

MONOSTATIC PATTERN — — — — — —
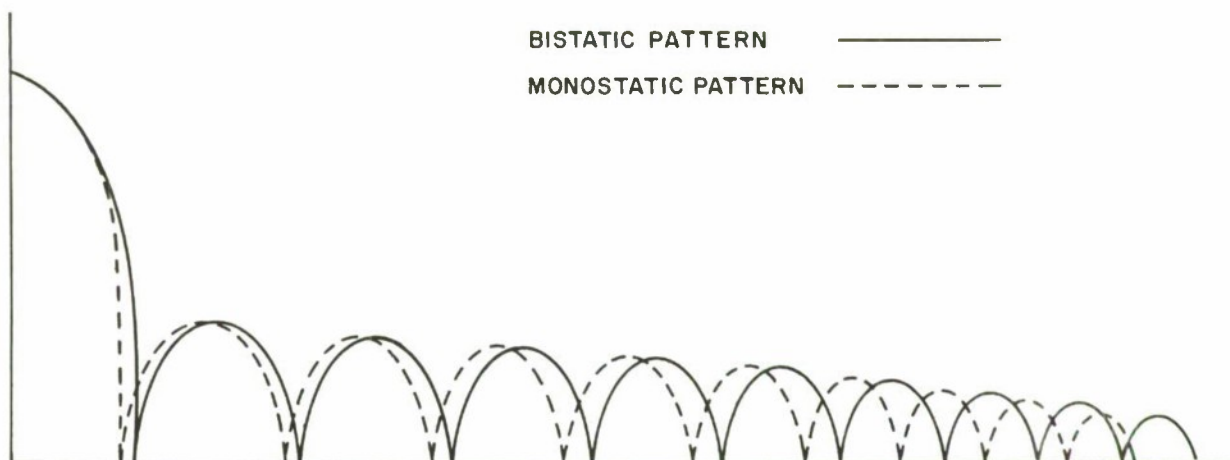
Figure 3.

the incident magnetic field on the surface in the illuminated region and zero elsewhere) are essentially the same as in the point scattering case just discussed. [1, 2]

SECTION II

DIPOLE POLARIZATION EFFECTS

The previous section considers the cases where scattering bodies do not depolarize. In order to study polarization effects on the monostatic-bistatic equivalence first consider an infinitesimal dipole oriented along the axes of a spherical polar coordinate. See Figure 4.

Figure 4.

The electric and magnetic fields are polarized in the $\hat{\theta}$ and $\hat{\phi}$ directions, respectively, and are given by

$$\bar{E} = \frac{j\omega\mu \, I_o \, h}{4\pi r} \, e^{jkr} \, \sin\theta \, \hat{\theta} \tag{9}$$

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$$\bar{H} = \sqrt{\frac{\epsilon}{\mu}} \; \hat{r} \times \bar{E} \quad , \tag{10}$$

where I_o is the current induced in the dipole of length h. If the receiving antenna is a unit dipole oriented in the θ_2, ϕ_2 plane with its axis given by
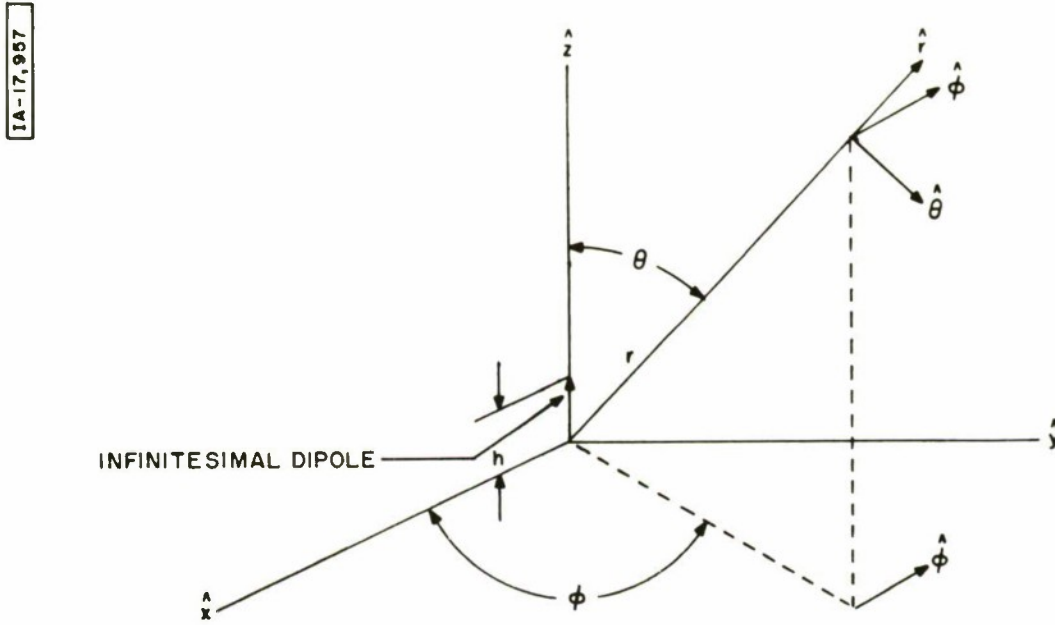
$$\hat{b}_2 = \hat{\theta}_2 \cos \eta_2 + \hat{\phi}_2 \sin \eta_2 , \tag{11}$$

then the current induced at the receiver is given by the projection of $\bar{E}$ on $\hat{b}_2$ or

$$I_1 = \frac{j \omega \mu I_o h}{4 \pi r_2} \; c^{\,jkr_2} \; \sin \theta_2 \cos \eta_2 \quad . \tag{12}$$

By reciprocity, the current induced in the dipole h for unit transmission vector with coordinates r_1, θ_1, and η_1 is given by

$$I_o = \frac{j \omega \mu h}{4 \pi r_1} \; e^{\,jkr_1} \; \sin \theta_1 \cos \eta_1 \quad . \tag{13}$$

Substituting Equation (13) into (12) gives an expression for the received current,

$$I_1 = A(r) \left(\sin^2 \theta_M - \sin^2 \xi' \right) \cos \eta_1 \cos \eta_2 \tag{14}$$

where, taking the scalar distances $r_1 = r_2 = r$,

$$A(r) = \left(\frac{j \omega \mu}{4 \pi r} \right)^2 e^{j2kr} \tag{15}$$

and

$$\theta_M = 1/2 \left(\theta_1 + \theta_2 \right) \tag{16}$$

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$$\xi' = 1/2 \left(\theta_1 - \theta_2 \right) . \tag{17}$$

From the monostatic propagation direction $\hat{r}$ defined in Equation (3), θ_M is the angle between $\hat{r}$ and the dipole axes $\hat{z}$, while ξ' is the projection of the bistatic angle onto the $\hat{r}$, $\hat{z}$ plane. Since the projection of the bistatic angle is always less than or equal to the angle, then it follows from Equation (14) that the maximum error introduced by assuming the monostatic-bistatic theorem is second order in the bistatic angle. Using the results of Section III, the theorem can be extended to a collection of parallel dipoles or to a dipole of finite length.

SECTION III

EXTENSION TO GEOMETRIC THEORY OF DIFFRACTION

The law of edge diffraction states that the angle of diffraction is equal to the angle of incidence.[3] Thus, the incident wave sets up a cone of diffracted waves at an angle β which is defined by

$$\cos \beta = I \cdot T = D \cdot T \quad , \tag{18}$$

where I, D, and T are unit vectors that define the directions of the incident wave, diffracted wave, and tangent, respectively. See Figure 5.
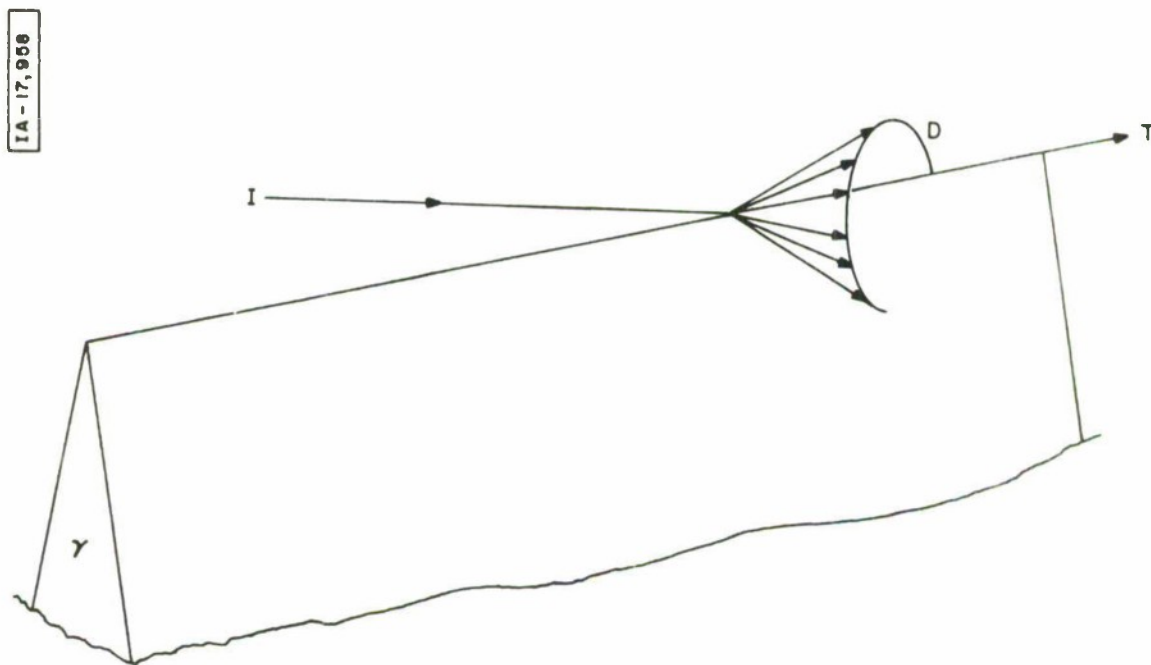
Figure 5.

The monostatic vector is defined by $M = I - D$. From Equation (18) it is clear that $M \cdot T = 0$, and M is the plane orthogonal to the wedge. See Figure 6.
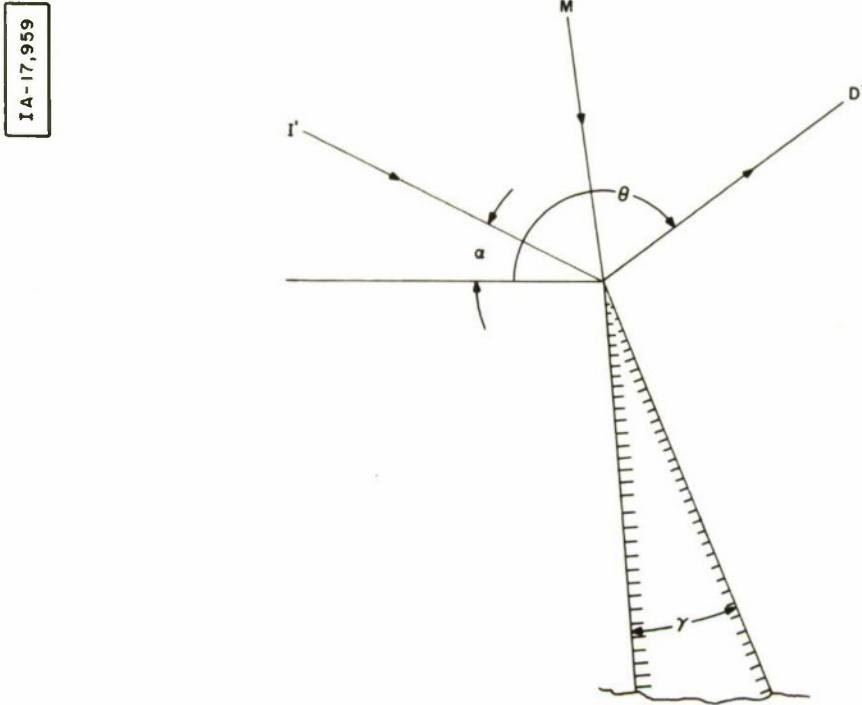
Figure 6.

Here I' and D' are the projections of I and D in the normal plane, and α and θ are the angles between the projected incidents and diffracted rays and the normal to one wedge face.

Sommerfeld's exact solution for diffraction of a plane wave by an infinite wedge consists of the incident and reflected waves of geometrical optics plus a third or "diffracted" term. When the third term is expanded asymptotically for large values of kr the following diffraction coefficients result: [3]

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$$d_+ = \frac{e^{j1/4\tau} \sin\frac{\tau}{q}}{q(2\tau k)^{1/2} \sin\beta} \left[\left(\cos\frac{\tau}{q} - \cos\frac{\theta - d}{q}\right)^{-1} \right.$$

$$\left. \mp \left(\cos\frac{\tau}{q} - \cos\frac{\theta + \alpha + \tau}{q}\right)^{-1} \right] , \qquad (19)$$

where β is the angle of incidence (or angle of diffraction) which is $1/2\,\tau$ in the monostatic case. The upper sign in Equation (19) applies for Dirichlet boundary conditions and the lower for Neumann type boundary conditions, (i.e., incident polarization parallel and perpendicular to the edge). The parameter q is given by

$$q = 2 - \frac{\gamma}{\tau} , \qquad (20)$$

where γ is the included wedge angle.

It is convenient to define the quantities $b_\pm$ as the following linear combination of $d_\pm$

$$b_\pm = 1/2 \left(d_+ \pm d_-\right) . \qquad (21)$$

Now from Equations (19) and (21) the ratio of b_- to b_+ is given by

$$\frac{b_+}{b_-} = \frac{2\left(\sin\frac{\tau}{2q}\right)^2}{\cos\frac{\tau}{q} - \cos\left(\frac{\theta + \alpha + \tau}{q}\right)} - \frac{2\left(\sin\frac{\theta - \alpha}{2q}\right)^2}{\cos\frac{\tau}{q} - \cos\left(\frac{\theta + \alpha + \tau}{q}\right)} . \qquad (22)$$

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From Equation (22) we see that this ratio is independent of the incidence angle. Furthermore, the first term represents the monostatic return for an angle of incidence of

$$\theta_M = 1/2 \ (\theta + \alpha) \ , \tag{23}$$

while the second term represents the error which is due to a projected bistatic angle of

$$\xi' = 1/2 \ (\theta - \alpha) \ . \tag{24}$$

The fractional error in assuming monostatic-bistatic equivalence is given by the ratio of these two terms or by

$$error = \left(\frac{\sin \frac{\xi'}{2q}}{\sin \frac{\pi}{2q}} \right)^2 . \tag{25}$$

It is interesting to note that the percentage of error depends only upon the projected bistatic angle and the wedge angle, which is represented in terms of q by Equation (20). For circular transmission the ratio of b_- to b_+ represents the ratio of the return with the same hand to that with the opposite hand as the transmitted polarization. Equation (25) then expresses the fractional error that is introduced in this ratio by assuming that the monostatic-bistatic equivalence theorem applies.

Figure 7 which illustrates this error in db as a function of bistatic angle for 0, 90, and 179 degree wedge angles shows that the error is less than .1 db for projected bistatic angles up to 10 degrees and less than .5 db for angles up to 30 degrees.
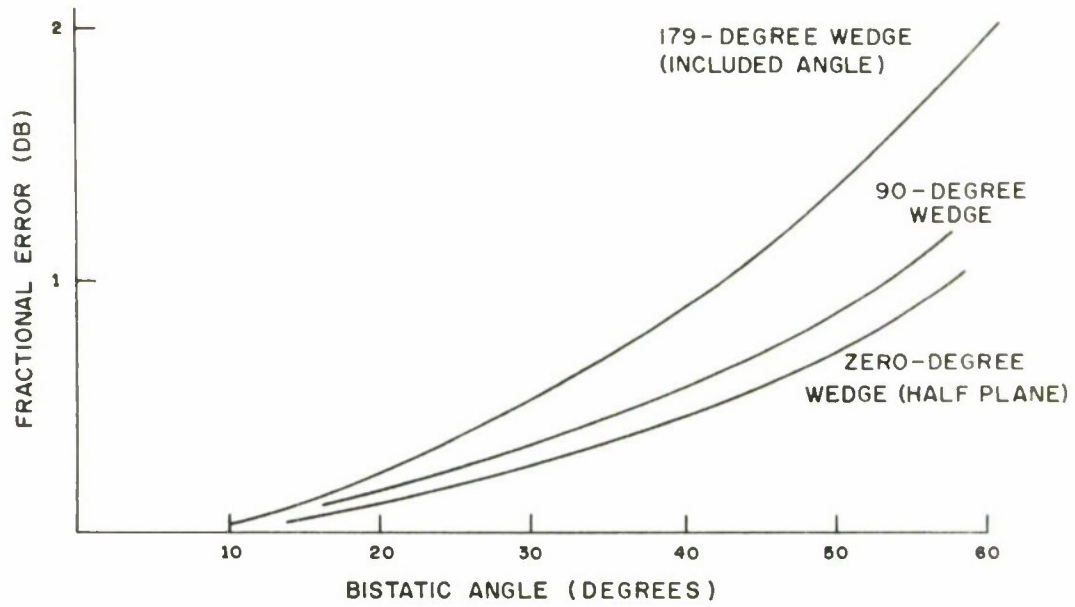
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Figure 7.

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SECTION IV

CONCLUSIONS

The monostatic-bistatic equivalence theorem can be made exact for non-interacting scattering centers and for those bodies for which the scattering can be described by physical optics, if the frequency of the equivalent monostatic antenna is given in terms of the bistatic angle by Equation (5),

$$\omega_M = \omega \cos \frac{\xi}{2} \; .$$

This causes a widening of the bistatic lobe pattern which is second order in the bistatic angle.

When the equivalence theorem is extended to depolarizing bodies, all of the error terms cannot be accounted for in a simple manner, such as a frequency change. However, in the case of dipole scatterers with parallel directions or acute wedges, the error in the equivalence approximation is again second order in the bistatic angle. For example, the error for acute wedges is less than one half a db for bistatic angles up to thirty degrees.

On the other hand, for such depolarizing bodies as a collection of dipoles with random orientation or obtuse wedges where multiple reflections occur, the theorem is no longer valid. For these reasons the theorem must be restricted to convex bodies where any depolarizing edges have a parallel orientation. A convex body of revolution, for example, satisfies these restrictions.

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2. Kell, Robert, "Monostatic-Bistatic Equivalence Theorem" (to be published Proc. IEEE, Electromagnetic Scattering Issue).

3. Lewis, R. M., Keller, J. B., Asymptotic Methods for Partial Differential Equations: The Reduced Wave Equation and Maxwell's Equations, New York University (January 1964), Research Report No. EM.-194.

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